TEMPERATURE DEPENDENCE OF THE ELASTIC CONSTANTS OF SODIUM 639

Temp. (°K)	Sample	С	C'	C_n	C11	C_{12}	A	ρ
78	1	5.78	0.680	13.26	8.15	6.79	8.50	1.010
78	2	5.77	0.695	13.34	8.26	6.87	8.30	1.010
115	1	5.53	0.665	12.85	7.98	6.65	8.31	1.006
115	2	5.53	0.673	12.94	8.08	6.74	8.21	1.006
155	1	5.27	0.650	12.41	7.78	6.48	8.11	0.999
155	2	5.27	0.652	12.50	7.88	6.58	8.09	0.999
195	1	5.00	0.634	11.97	7.61	6.34	7.89	0.991
195	2	4.99	0.630	12.07	7.71	6.45	7.92	0.991
300	Daniels	4.19	0.585	11.01	7.41	6.24	7.16	0.970

 Table 1. Values of the adiabatic elastic constants, the elastic anisotropy, and the density of sodium.

 Elastic constants in units of 10¹⁰ dynes-cm⁻² and the density in g-cm⁻³

Table 2.	Values	of the	adiaba	tic and	isothermal bulk
modul	li of sou	dium in	units	of 1010	dynes-cm ⁻²

Temp. (°K)	$B_s(a)$	$B_T(b)$	$B_T(c)$
78	7.30	7.29	7.09
115	7.14	7.10	6.99
155	6.97	6.74	6.92
195	6.82	6.50	6.72
300 (Daniels)	6.61	6.16	6.33

(a) $B_s = C_n - C - C'3$.

(b) Computed from B_{δ} .

(c) Swenson.

from SWENSON⁽¹⁰⁾ and the values of C_p were obtained from MARTIN.⁽¹¹⁾ Swenson's values of B_T are also listed and the agreement is well within experimental error of either measurement.

The values of the adiabatic elastic constant C_{11} obtained from direct measurement of the longitudinal wave velocity on a crystal oriented along [100] are shown in Table 3. The values of C_{11} obtained in this manner agree well with indirect determination of this quantity.

Table 4 includes the experimental values of the shear constants C_{44} , C' and the elastic anisotropy at 78°K along with FUCHS'⁽³⁾ theoretical values at 0°K. The theoretical values take into account only the electrostatic contribution to the elastic constants. Also included in the tables are the experimental values of these quantities at 80°K determined by QUIMBY and SIEGEL⁽¹⁾ and the experimental values of BENDER⁽²⁾ at 90°K. Daniels' values at 300°K are also listed.

The values of the elastic constants obtained in this work have a calculated precision of 2-3% when one includes errors in length measurements, transit time, and density. The internal consistency appears to be better than this, as can be seen in Figs. 1 and 2.

Table 3. Values of the adiabatic elastic constant C₁₁ obtained from velocity measurements in [100] direction

Temp. (°K)	C11		
78	8.46		
115	8.29		
155	8.11		
195	7.93		
300 (Daniels)	7.38		

The elastic constants C_{44} , C' and the elastic anisotropy plotted as a function of temperature are shown in Fig. 1. Daniels' values of these quantities at 300°K and the data of Quimby and Siegel at 80°K and 200°K are also shown. Since sodium undergoes a low temperature martensitic transformation from a b.c.c. structure to an h.c.p. structure, ⁽¹²⁾ no low temperature data were obtained. The values of the elastic constants at 0°K can be obtained by extrapolating the present data and breaking the curve at about 20°K so that the slope is zero at 0°K.

Temp. (°K)	C_{44}	<i>C</i> ′	A	Source
0	5.32	0.715	7.44	Theory
78	5.78	0.688	8.40	Present
80	5.93	0.725	8.18	Quimby and Siegel
90	6.18	0.830	7.45	Bender
300	4.19	0.585	7.16	Daniels

Table 4. Comparison of the shear constants C_{44} , $C' = 1/2(C_{11}-C_{12})$, and the elastic anisotropy. Elastic constants in units of 10^{10} dyn-cm⁻²



FIG. 1. The elastic constants

 $C = C_{44}$ and $C' = 1/2(C_{11}-C_{12})$ and the elastic anisotropy A = C/C' plotted as a function of temperature. Elastic constants in units of 10^{10} dynes-cm⁻².

The values of the shear constants obtained in the present study are in substantial agreement with the values of these quantities obtained by Quimby and Siegel. This experiment, like that of Quimby and Siegel yields more accurately the temperature variation of the elastic constants rather than the absolute value of the elastic constants themselves. The agreement here is excellent as can be seen in Fig. 1. This experiment also yields values of C_n , C_{11} , and B_s and their temperature dependence, which were either unknown or unreliable before the present study was completed. Figure 2 shows the elastic constants C_n , C_{11} , and the adiabatic bulk modulus as a function of temperature. The C_{11} data is taken from measurements on an acoustic specimen oriented along [100].



FIG. 2. The elastic constants C_n , C_{11} and B_s plotted as a function of temperature. Units are 10^{10} dynes-cm⁻².

The observed temperature coefficients at constant pressure of the elastic constants were obtained from Figs. 1 and 2; these are related to the pressure variation of the elastic constants by the thermodynamic relation,

$$\left(\frac{d\ln C}{dT}\right)_{P} = \left(\frac{d\ln C}{dT}\right)_{V} + \alpha \left(\frac{d\ln C}{d\ln r}\right)_{T} \quad (3)$$

where α is the coefficient of linear expansion at 300°K and the quantity $(d \ln C/d \ln r)_T$ can be obtained from the pressure measurements of